A Finite-volume Method for On-package IR Drop Characterization

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Abstract: A control-volume numerical method is developed for package IR drop analysis. By meshing conductors and solving the governing equations using a finite-volume method with multigrid linear solver, steady-state volumetric currents resulted based IR drop can be accurately and efficiently computed for power delivery analysis.

1. Introduction

IR drop is the voltage drop due to the finite conductivity of the power distribution network. IR drop, starting from the power supply source (VRM module) to the IC circuits, experiences three stages: on-board IR drop, on-package IR drop and on-chip IR drop [1]. As technologies advance the power dissipation in higher density ICs, tolerable IR drop decreases significantly due to lower supply voltages. On-board IR drop can be ignored because large planar structures, (which reduces the power dissipation by distributing currents in the plane) are used for power distribution in PCB. However, on-package IR drop, as part of the IR drop in the power distribution network, becomes as important as the on-chip IR drop for current and future IC design engineers due to the decrease of supply voltages and tolerable voltage fluctuation.

IR drop is usually referred to DC IR drop. At low frequency, the inductance effect may be negligible in an electrically small package. Thus, resistance determines the current flow path, and the current flowing in a conductor is uniformly distributed over the whole cross-sectional area of the conductor. In this case, a steady state volumetric current-based voltage drop is equivalent to the DC IR drop.

To characterize the on-package IR drop accurately, a numerical method based on volume discretization is needed for solving the current distribution in a relatively complex geometry with small geometric details, such as via holes, copper islands, wires and via pads included.

In this paper, we first derive the steady-state IR drop mathematical model─Laplace's equation. Then, an introduction to the finite volume method is given and the numerical discretization of Laplace's equation is derived for IR drop modeling. Next, to illustrate the effectiveness of the approach for a complex power delivery system, we simulate an 8-layer Altera flipchip package with a numerical tool based on this approach. The accuracy and efficiency of this numerical method is demonstrated by comparing the simulation results to measurements and other available resources.

2. IR Drop Governing Equations: Ohm's Law and Laplace's Equation

When there are currents flowing in a conductor at steady state, the governing equations for steady current density J in the absence of energy sources are [2]

$$\nabla \cdot \mathbf{J} = 0 \qquad (1)$$

If we now also restrict our consideration to linear, isotropic, homogenous conductors with a uniform volumetric current, from the Ohm's law,

$$\mathbf{J} = \sigma \mathbf{E} \qquad (2)$$

At steady-state, the electric field can be expressed as a gradient of a scalar electric potential

$$\mathbf{E} = -\nabla \phi \qquad (3)$$

which leads to Laplace's equation of scalar electric potential

$$\nabla^2 \phi = 0 \qquad (4)$$

On a conductor's surface, the normal **E** field has to be continuous, therefore, in a source free case,

$$\frac{\partial \Phi}{\partial n} = 0 \qquad (5)$$

To solve for a steady-state current flowing in a conductor, we inject a known current at one end and ground the other end. Thus, $\frac{\partial \phi}{\partial n} = \frac{\mathbf{J}}{\sigma}$, $\phi = 0$ at the current injection (source) and grounding ends (sink), respectively. To solve Laplace's equation given these boundary conditions, a finite-volume based numerical method is used.

3. Finite-volume Method Formulation

The finite-volume method has been widely applied to computational fluid dynamics (CFD). Unstructured cell based finite-volume schemes are advanced schemes which can collocate the pressure-velocity on cells and nodes. The conservation law is enforced on the basic cell [3][4]. In this section, we extend the finite-volume method to the source-free Laplace's equation which describes the steady-state DC current flow in a conductor.

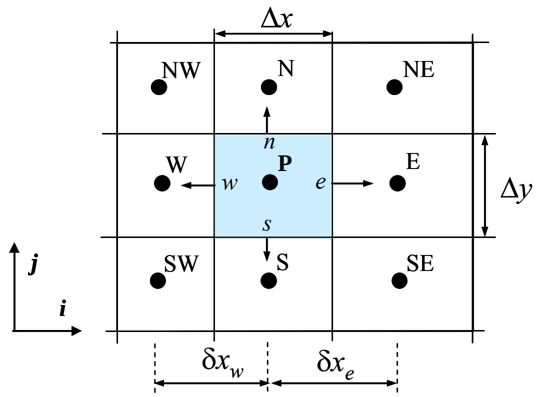


Figure 1. Two-dimensional control volume.

The arrangement of cells under consideration is illustrated in Figure 1. We focus on cell P and its neighbors, the cells E, W, N and S. Discrete values of ϕ are stored at cell centroids. We also store the conductivity at cell centroids. The cell faces e ,w ,n and s are associated with area vectors $\mathbf{A}e$, $\mathbf{A}w$, $\mathbf{A}n$ and $\mathbf{A}s$. The vectors are positive pointing outwards from the cell P. The volume of the cell P is $\Delta V = \Delta x \times \Delta y$. By integrating equation (1) over the cell P:

$$\int_{\Delta v} \nabla \cdot \mathbf{J} dV = 0 \qquad (6)$$

Next, we apply the divergence theorem to yield

$$\int_{A} \mathbf{J} \cdot d\mathbf{A} = 0 \qquad (7)$$

The surface integral represents the integral over the control surface A of the cell. Assuming that current flux (current density) varies linearly over each face of the cell P so that it may be represented by its value at the face centroid,

$$\sum_{f=e,w,n,s} \mathbf{J}_f \cdot \mathbf{A}_f = 0 \qquad (8)$$

where the surface vector $\mathbf{A}$ can be written as

$$\mathbf{A}_e = \Delta y \hat{x}, \mathbf{A}_w = -\Delta y \hat{x}, \mathbf{A}_n = \Delta x \hat{y}, \mathbf{A}_s = -\Delta x \hat{y} \qquad (9)$$

To complete the discretization process, we assume that the voltage potential ϕ varies linearly between cell centroids. Thus, from (3) we may write

$$\mathbf{J}_e \cdot \mathbf{A}_e = -\sigma_e \Delta y \frac{\phi_e - \phi_p}{(\delta x)_e} \qquad (10)$$

Similar expressions can be written for the rest of fluxes on the other surfaces. Substituting (9) ,(10) into (8) yields a discrete equation for ϕ_p:

$$a_p\phi_p = a_e\phi_e + a_w\phi_w + a_n\phi_n + a_s\phi_s \qquad (11)$$

Where,

$$a_e = \frac{\sigma_e \Delta y}{(\delta x)_e}, a_w = \frac{\sigma_w \Delta y}{(\delta x)_w}, a_n = \frac{\sigma_n \Delta x}{(\delta y)_n}, a_s = \frac{\sigma_s \Delta x}{(\delta y)_s}, a_p = a_e + a_w + a_n + a_s$$

A more general formulation for a 3D case is

$$a_p\phi_p + \sum_{nb} a_{nb}\phi_{nb} = 0 \qquad (12)$$

Here, the subscript *nb* denotes the cell neighbors. For equation (12) on unstructured mesh, the number of cell neighbors is also arbitrary. Consequently, familiar line-iterative solvers cannot be used. Instead, the system is solved using an algebraic multigrid procedure. The advantage of this algebraic multigrid procedure is that discretization of the governing equations at the coarse mesh levels is avoided which lead to an optimum linear solver performance for the equation [5].

4. Package IR Drop Analysis Using the Finite-volume Method

For package IR drop analysis, a steady current is injected at the input and a reference electric potential is applied at the output of a power/ground net. Therefore, the current distribution and the potential on the power/ground nets can be solved without any geometry simplifications. To demonstrate the superiority of the numerical method, we take the VCC net of an Altera 8-layer flipchip package shown in Figure 2, apply source (current) at the solder bumps and sink (ground) at the solder balls [6]. The computed results of resistance from individual solder bump to individual solder ball are listed in Table 1. A very good correlation has been shown between the measured resistance and the simulated results. In this case, the average relative error is less than 5%.

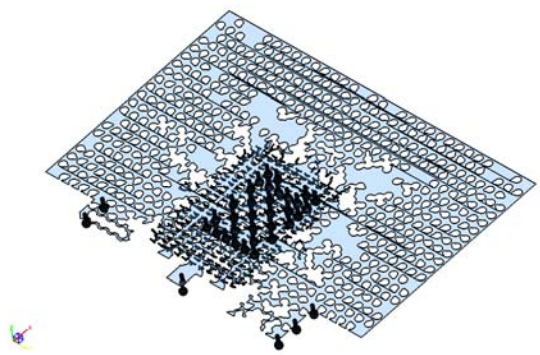
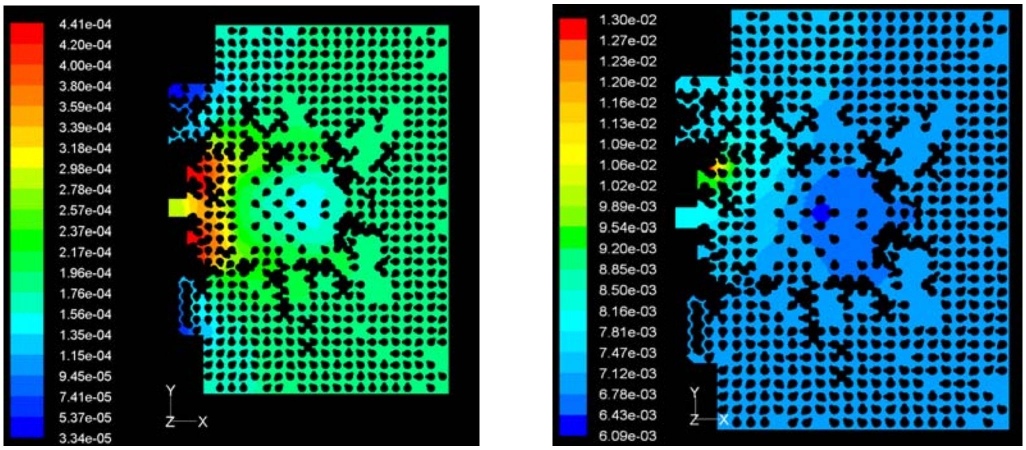


Figure 2. A power net of an Altera flipchip package.

Table 1. The computed resistance of the power nets by alternating the source and sink

Bump	Ball	Resistance (mOhm) Simulated	Resistance (mOhm) Measured[1]
I23	AG10	34.4	32.7
I23	T19	24.4	26.7
Q12	AG10	26.4	32.5
Q12	T19	16.0	16.5
AG11	AG10	25.7	24.9
AG11	T19	16.4	18.9
AY0	AG10	31.7	31.8
AY0	T19	22.8	25.8



(a). Current from all bumps to all Balls with all bumps excited and balls grounded

(b). Current from bump I23 to Ball AG10 with only one bump excited and one ball grounded.

Figure 3. Contour plots of voltage (electric potential) on VCC net.

Contour plots of electric potential on the power plane of the VCC net are shown in Figure 3(a) and (b) with different excitations: in plot (a), all bumps are connected to the current source and all balls are grounded, which results a relatively well distributed currents on the VCC plane and the electric potential on the plane is also well distributed. The maximum voltage drop between the plane and grounded balls is 4.41E-4V; in plot (b), only one bump is excited and one ball grounded, the electric current density is highly concentrated along the current flow path which is from bump I23 to ball AG10, which results in a high potential concentration right on the path. As a result, the maximum voltage drop 1.30E-2V, from the plane to the grounded ball AG10, is much higher than the voltage drop in (a).

For the discussed cases, it took about 43 minutes and 500 MB Memory on a Pentium 4 2.8GHz, 2GB Memory PC.

5. Conclusion

In this paper, an efficient, state-of-art numerical technique: finite-volume method with multigrid acceleration is applied to package IR drop characterization. This volume based method calculates the steady-state DC current in the conductors by discretizing the conductor's three-dimensional geometry without any geometric simplification. The solution procedure delivers an accurate solution for a complex package structure much faster than any other conventional numerical methods.

6. Acknowledgement

The authors would like to thank Dr. Anil Pannikkat from Altera Corp. for providing package model and measured data for comparison and validation.

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