Conduction in Series

Abstract—*In this exercise, one-dimensional heat conduction in series is modeled. The solid is represented in two dimensions. The length of the two solids is specified. Coarse, medium, and fine meshes are available. The thermal conductivity of the two solids can be specified within limits. Temperature or heat flux at the left wall, and temperature at the right wall can be specified. Reports of heat flux, temperature gradient, and solid interface temperature are available. A plot of temperature versus axial position can be displayed. A temperature contour plot is also available.*

1 Introduction

Thermal conduction is an important mode of heat transfer. Fourier's law of heat conduction relates the heat transfer rate to the temperature gradient, where thermal conductivity is represented by a constant of proportionality.

2 Modeling Details

The solid is represented in two dimensions by a rectangle. The procedure for solving the problem is as follows:

1. Create the geometry.
2. Set the solid properties and boundary conditions.
3. Mesh the domain.

FlowLab creates the geometry and mesh, and exports the mesh to FLUENT. The boundary conditions and flow properties are set through parameterized case files. FLUENT continues to solve the problem until the convergence limit is satisfied or the specified number of iterations is achieved.

2.1 Geometry

The geometry is created from a set of four vertices. Edges are created over the vertices and are stitched with faces to facilitate meshing. The geometry consists of four walls and an interface (Figure 2.1).

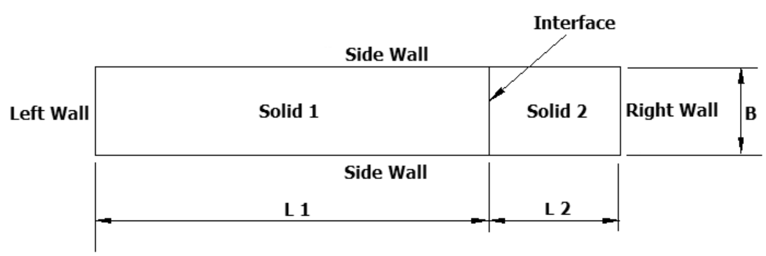

Figure 2.1: Schematic of the Solid with Boundaries

You have to specify the lengths of the two solids (L1 and L2).

2.2 Mesh

Coarse, medium, and fine mesh types are available. The discretization scheme used is based on the following logic:

- `$S1` = Number of cells applied along length L1 or L2 depending upon which is larger.
- `$S2` = If L1 > L2, $S2 can be determined (as shown in Table 2.1). If L2 > L1, the length ratio in the table would be inverted.
- `$S3` = Number of cells along the breadth, B.

The numerical values for the two discretization parameters are given in Table 2.1.

Mesh Type	$S1	$S2	$S3
Coarse Mesh	10	(L2/L1)×$S1	3
Medium Mesh	20	(L2/L1)×$S1	4
Fine Mesh	40	(L2/L1)×$S1	5

Table 2.1: Mesh Discretization Logic

The face is meshed using the map scheme after the edges are discretized into intervals (Figure 2.2).

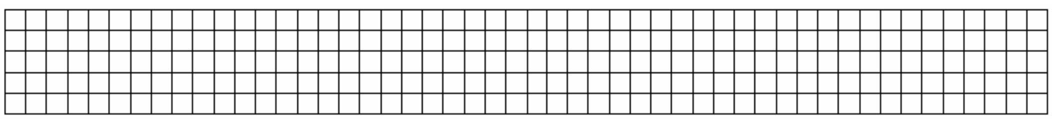

Figure 2.2: Mesh Generated by FlowLab

2.3 Physical Models for FLUENT

The energy equation is solved in the solid domain. Since there is no fluid flow, mass and momentum equations are not solved.

2.4 Material Properties

The default solid materials are Aluminum and Copper. The thermal conductivity of the solids can be specified within the limits. Other materials such as Steel, Wood and a User Defined solid can also be selected.

2.5 Boundary Conditions

You can specify the following boundary conditions:

- Right wall temperature
- Left wall temperature *
- Left wall heat flux *

* *Based on the thermal condition (heat flux or temperature specification at the left wall).*

The following boundary conditions are assigned in FLUENT:

Left Wall	Wall
Right Wall	Wall
Interface between the solids	Interior
Side Walls	Symmetry

Table 2.2: Boundary Conditions Assigned to FLUENT

2.6 Solution

The mesh is exported to FLUENT along with the physical properties and the initial conditions specified. The material properties and the initial conditions are read through the case file. Instructions for the solver are provided through a `journal` file. Once the solution is converged or the number of specified iterations is met, FLUENT exports data to a `neutral` file and to `.xy plot` files. GAMBIT reads the `neutral` file for postprocessing activities.

3 Scope and Limitations

The maximum temperature allowed by FLUENT (hence FlowLab as well) is 5000 K. If the temperature exceeds this limit, it will be artificially restricted to 5000 K. Hence, the results obtained for cases where the temperature exceeds this limit may not be correct.

Difficulty in obtaining convergence or poor accuracy may result if input values are used outside the upper and lower limits suggested in the problem overview.

4 Exercise Results

4.1 Reports

The following reports are available:

- Heat flux
- Average temperature gradient
- Interface temperature between the two solids

4.2 XY Plots

The plots reported by FlowLab include:

- Residuals
- Temperature distribution

Figure 4.1 presents temperature versus axial position.

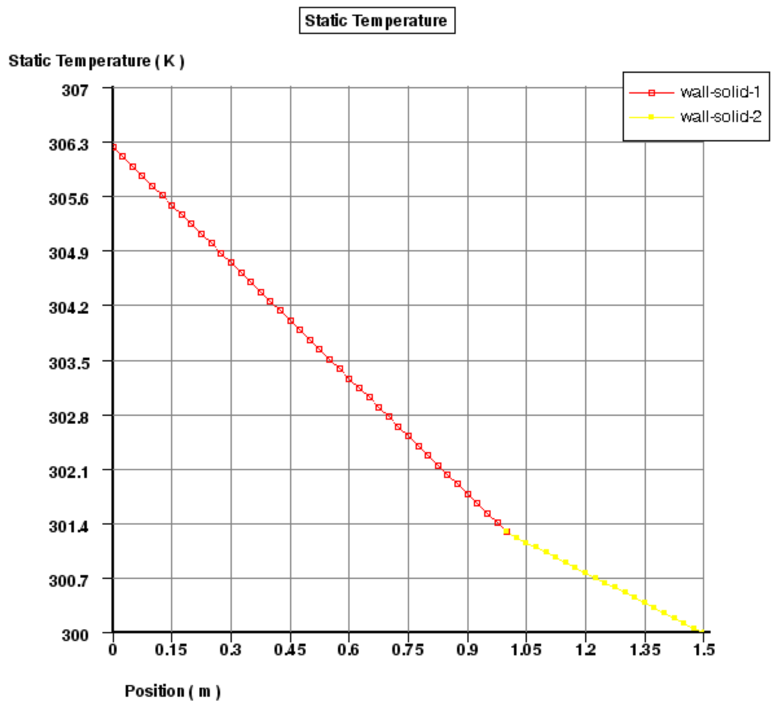

Figure 4.1: Temperature Distribution along Two Solids

4.3 Contour Plots

A contour plot of temperature is available (Figure 4.2).

Figure 4.2: Contour Plot of Temperature

5 Verification of Results

The net thermal resistance for conduction in series is given by:

$$\sum R = R_A + R_B = \frac{L_A}{k_A A} + \frac{L_B}{k_B A} \tag{5-1}$$

The heat rate is given by:

$$q_x = \frac{\Delta T}{\sum R} = \frac{(T_1 - T_3)}{\sum R} \tag{5-2}$$

where, T_3 is the temperature at the end of block (the right wall). The net heat flux is given by the heat rate divided by the total area.

The interface temperature T_2 can be calculated using the following relation:

$$T_2 = \frac{\frac{k_2 L_1}{k_1 L_2} T_3 + T_1}{1 + \frac{k_2 L_1}{k_1 L_2}} \tag{5-3}$$

The following default settings are used to verify the CFD model:

$L_1 = 1\ m$
$L_2 = 0.5\ m$
$A_A = 0.1\ m^2$
$A_B = 0.1\ m^2$
$k_A = 202.4\ W/(m-K)$
$k_B = 387.6\ W/(m-K)$
$T_3 = 300\ K$

Heat flux values computed using the default settings summarized above are compared with theoretical values in Table 5.1. FlowLab results were generated using the fine mesh option and a convergence limit of `1e-12`.

Left Wall Temperature (K)	**Heat Flux (W/m^2)**		**Interface Temperature(K)**	
	FlowLab	**Theory**	FlowLab	**Theory**
400	16049.6	16050	320.941	321
500	32099.1	32099	341.881	341
1000	112347	112347	446.584	445
4000	593834	593834	1074.8	1067

Table 5.1: Heat Flux Verification

6 Sample Problem

1. Run the cases using the default options for the exercise and all the three mesh types. It can be observed that the temperature gradient is slightly influenced by mesh density. Compare the results obtained using FlowLab with theoretical results.

2. Change the left wall boundary condition to the *wall temperature* type and run additional cases using the same default settings and all three mesh options. Again, compare FlowLab results with theoretical results.

3. Vary the thermal conductivity of the two solids and observe resulting interface temperature changes.

7 Reference

[1] *Incropera, F. P., and DeWitt, D. P.*, "Fundamentals of Heat and Mass Transfer", 4^{th} Ed., Ch. 3.