Flow Over a Heated Plate

Abstract—*In this exercise, boundary layer development over a plate is modeled. The length and height of the plate are specified as inputs. Coarse, medium, and fine mesh types are available. The free-stream velocity and temperature of the fluid can be specified. Fluid density, viscosity, thermal conductivity, and specific heat can be varied. Mass averaged velocity at the outlet, wall friction force acting on the plate, total wall heat transfer, and temperature change across the plate are reported. Plots of skin friction coefficient, Nusselt number distribution, velocity profiles, and temperature profiles are available. Contours of stream function, temperature, turbulent intensity and dissipation rate, and velocity magnitude can be displayed. A velocity vector plot is also available.*

1 Introduction

The boundary layer plays an important role in heat transfer. A boundary layer is formed whenever there is a viscous flow over a solid surface. Thickness of the boundary layer is important with respect to drag forces and heat transfer between the fluid and the surface.

Using this exercise, it is possible to evaluate the thickness of a boundary layer across the surface of a plate, as a function of Reynolds number. Comparisons can be made between laminar and turbulent flow cases. The thickness of the hydrodynamic boundary layer and the thermal boundary layer can be compared.

2 Modeling Details

The fluid region is represented in two dimensions. The procedure for solving the problem is:

1. Create the geometry.
2. Set the material properties and boundary conditions.
3. Mesh the domain.

FlowLab creates the geometry and mesh, and exports the mesh to FLUENT. The boundary conditions and flow properties are set through parameterized case files. FLUENT converges the problem until the convergence limit is met or the number of iterations specified by the user is achieved.

2.1 Geometry

The flow domain is shown in Figure 2.1. To avoid a sudden start to the boundary layer, a symmetry boundary with length 0.1×L is created before the plate as shown in the Figure 2.1. Connecting edges between the upper and lower edges are created and stitched into separate faces.

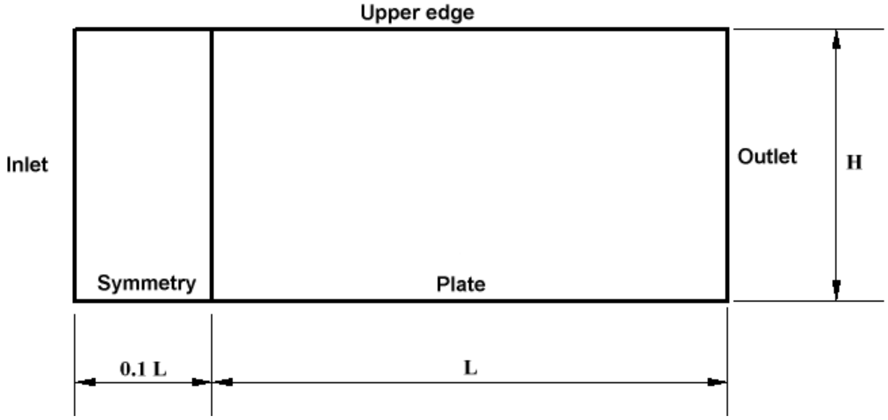

Figure 2.1: Schematic of the Plate

2.2 Mesh

Coarse, medium, and fine mesh types are available. Mesh density varies based upon the assigned Refinement Factor. The Refinement Factor values for the mesh densities are given in Table 2.1.

Mesh Density	**Refinement Factor**
Fine	1
Medium	1.6
Coarse	2.56

Table 2.1: Refinement Factor

First Cell Height is calculated using the Refinement Factor, as follows:

$$First\ Cell\ Height = Refinement\ Factor \left[\frac{Yplus \times Log(0.06 \times Re) \times Viscosity}{0.477 \times Velocity \times Density} \right] \tag{2-1}$$

Reynolds number is used to determine Yplus, where Yplus values for turbulent flow conditions are summarized in Table 2.2.

Reynolds Number	**Flow Regime**	**Yplus/First Cell Height**
Re $\leq$ 1.0e+5	Laminar	First Cell Height = Domain height/1000
1.0e+5 $<$ Re $\leq$ 1.0e+6	Turbulent, Enhanced Wall Treatment	Yplus $<$ 10
Re $>$ 1.0e+6	Turbulent, Standard Wall Functions	Yplus $>$ 30

Table 2.2: Flow Regime Vs. Reynolds Number

The number of intervals along each edge is determined using geometric progression and Equation 2.2.

$$Intervals = INT\left[\frac{Log\left\{\frac{Edge\_length \times (Growth\_ratio - 1)}{First\ Cell\ Height} + 1.0\right\}}{Log(Growth\_ratio)}\right] \tag{2-2}$$

The edges are meshed using the First Cell Height and the calculated number of intervals. The entire domain is meshed using a map scheme. The resulting mesh is shown in Figure 2.2.

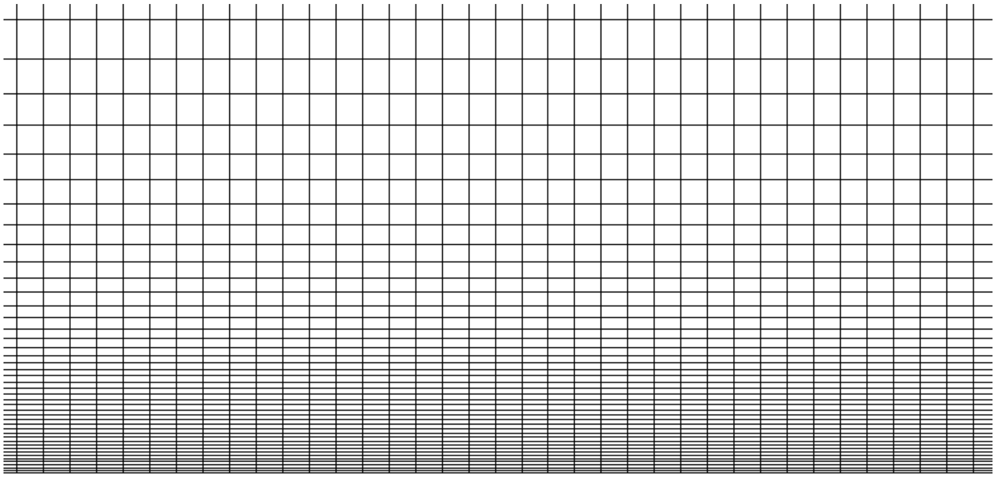

Figure 2.2: Mesh Generated by FlowLab

2.3 Physical Models for FLUENT

Based on the Reynolds number, the following physical models are recommended:

Re $\leq$ 1.0e+5	Laminar flow
1.0e+5 $<$ Re $\leq$ 1.0e+6	$k - \epsilon$ model
Re $>$ 1.0e+6	$k - \epsilon$ model

Table 2.3: Turbulence Models Based on Pipe Reynolds Number

If turbulence is selected in the Physics form of the Operation menu, the appropriate turbulence model and wall treatment is applied based upon the Reynolds number.

2.4 Material Properties

The default material is air. The following material properties can be specified:

- Density
- Viscosity
- Thermal conductivity *
- Specific Heat *

* *To be specified only when the heat transfer is on.*

Other materials such as Glycerin, Water, and a User Defined fluid can also be selected.

2.5 Boundary Conditions

The following boundary conditions can be specified:

- Inlet fluid velocity
- Inlet fluid temperature **
- Plate Wall temperature **
- Wall roughness *

** *To be specified only when the heat transfer is on.*

* *To be specified only when the flow is modeled as turbulent.*

The following boundary conditions are assigned in FLUENT.

Boundary	**Assigned As**
Inlet	Velocity inlet
Outlet	Pressure outlet
Plate	Wall
Upper edge	Pressure outlet

Table 2.4: Boundary Conditions Assigned in FLUENT

2.6 Solution

The Reynolds number and the Prandtl number are calculated based on the boundary conditions and material properties specified. Profiles of velocity and temperature are plotted at specified `x/L` locations, where `x` is the axial position and `L` is total plate length.

The mesh is exported to FLUENT along with the physical properties and the initial conditions specified. The material properties and the initial conditions are read through the case file. Instructions for the solver are provided through a `journal file`. When the solution is converged or the specified number of iterations is met, FLUENT exports data to a `neutral` file and to `.xy` plot files. GAMBIT reads the `neutral` file for postprocessing.

3 Scope and Limitations

The flow is transitional in the Reynolds number range of 1.0e+5 to 5.0e+5. In order to predict the correct flow and temperature field, enhanced wall treatment has been used. However, the accuracy of predicted results may vary by more than ten percent from experimental correlation in this region.

Difficulty in obtaining convergence or poor accuracy may result if input values are used outside of the upper and lower limits suggested in the problem overview.

4 Exercise Results

4.1 Reports

The following reports are available:

- Mass averaged outlet velocity
- Wall friction force
- Heat flux on the wall **
- Temperature change **

4.2 XY Plots

The plots reported by FlowLab include:

- Residuals
- Wall skin friction distribution
- Profiles of X velocity at the specified locations
- Boundary layer thickness
- Wall Nusselt number distribution **
- Profiles of temperature at the specified locations **
- Thermal boundary layer distribution **
- Wall Yplus distribution *

** *Available only when the heat transfer is on.*

* *Available only when the flow is modeled as turbulent.*

Figures 4.1 and 4.2 represent the skin friction distribution and Nusselt number distribution along the surface of the plate.

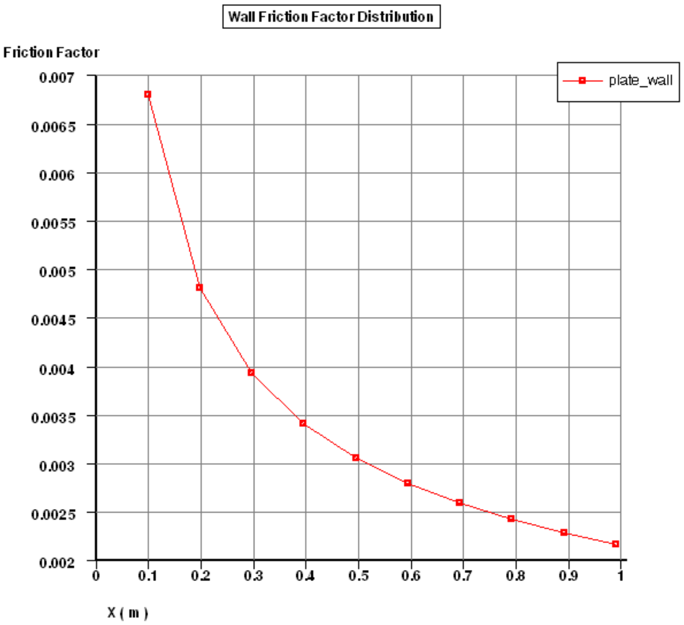

Figure 4.1: Wall Friction Factor Vs. Axial Position for Re = 1.0e+5

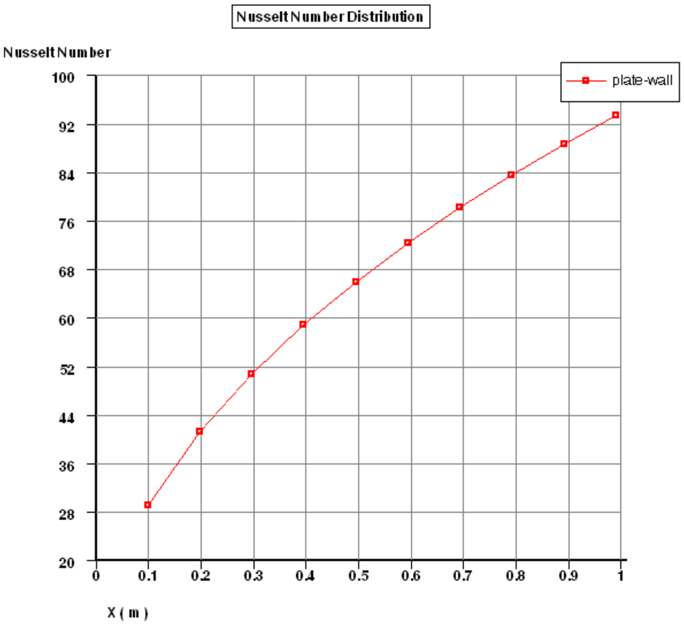

Figure 4.2: Nusselt Number Vs. Axial Position for Re = 1.0e+5

4.3 Contour Plots

Contours of velocity magnitude, x-velocity, y-velocity, turbulence intensity and dissipation rate, stream function, and temperature can be displayed. Figure 4.3 presents contours of velocity.

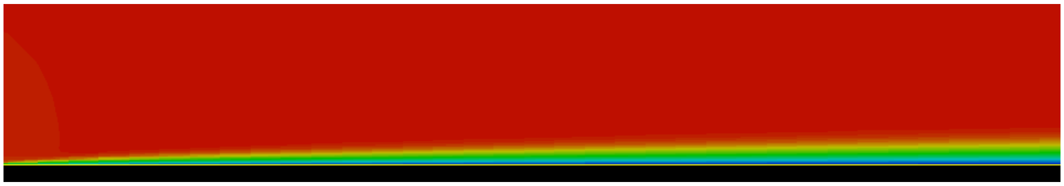

Figure 4.3: Contours of Velocity

5 Verification of Results

The results presented in Tables 5.1 and 5.2 were obtained using the fine mesh option, default material properties, and the default boundary condition for temperature. Inlet velocity and plate length were adjusted to obtain the required Reynolds number. Tables 5.1 and 5.2 provide a comparison of predicted skin friction factor and Nusselt number versus experimental correlation.

Re based on plate length	**Skin Friction Coefficient** (FlowLab)	**Skin Friction Coefficient (Correlation [1])**
1.0e+4	6.479e-3	6.64e-3
1.0e+5	2.088e-3	2.10e-3
1.0e+6	3.655e-3	3.75e-3
1.0e+7	2.58e-3	2.57e-3
1.0e+8	1.919e-3	1.87e-3

Table 5.1: Predicted Skin Friction Coefficient Vs. Correlation

Re based on plate length	**Nusselt Number (Correlation [2])**	**Nusselt Number** (FlowLab)
1.0e+4	29.48	29.44
1.0e+5	93.22	94.05
1.0e+6	1.66e+3	1.68e+3
1.0e+7	1.04e+4	1.10e+4
1.0e+8	6.60e+4	8.15e+4

Table 5.2: Predicted Nusselt Number Vs. Correlation

The correlations used for comparison with predicted results are as follows:

For Laminar Flow	**For Turbulent Flow**
Reynolds number < 1.0e+5	Reynolds number > 5.0e+5
$C_{f,x} = \frac{0.664}{\sqrt{Re_x}}$	$C_{f,x} = \frac{0.455}{ln^2(0.06 Re_x)}$
$Nu_x = 0.332 \times \sqrt{Re_x} \times (Pr)^{1/3}$	$Nu_x = 0.0296 \times (Re_x)^{4/5} \times (Pr)^{1/3}$

where,

$C_{f,x}$ = local wall friction coefficient
Nu_x = local Nusselt number
Re_x = local Reynolds number
Pr = Prandtl number

6 Sample Problems

1. Using the default settings (Reynolds number of 10^5, air, laminar flow), run the case and compare the results with theoretical values.
2. Run additional cases while varying the inlet velocity up to a Reynolds number of 1.0e+8.
3. Observe how the distribution of Nusselt number and skin friction coefficient changes as a function of Reynolds number. In addition, check the effect of Reynolds number on velocity and thermal boundary layer thickness.
4. Set the material properties to represent water, and perform Problem 3 again evaluating a Reynolds number range of 10^3 to 10^8.

7 Reference

[1] *White, Frank M.*, "Viscous Fluid Flow", International Edition, McGraw-Hill, 1991.

[2] *Incropera, F. P., and DeWitt, D. P.*, "Fundamentals of Heat and Mass Transfer", 4^{th} Ed., Ch. 7.

[3] *Schlichting, H.*, "Boundary-Layer Theory", 7th Edition, McGraw-Hill, 1979.