Heat Transfer through Fins

Abstract—*In this exercise, heat transfer through various fin geometries is modeled. Geometry configurations such as rectangular, trapezoidal, triangular, cylindrical, and parabolic profiles are available. The length, base thickness, and end thickness of the fin can be specified. Coarse, medium, and fine mesh types are available. Thermal conductivity of the fin material can be specified. Constant temperature or uniform heat flux boundary conditions can be applied at the base of the fin. Fully insulated or convective boundary conditions can be applied at the tip of the fin. The exercise reports base wall temperature, total area for heat convection, heat dissipation rate, fin efficiency, and fin effectiveness. Contours of temperature can be displayed.*

1 Introduction

Fins are used to enhance convective heat transfer in a wide range of engineering applications, and offer a practical means for achieving a large total heat transfer surface area without the use of an excessive amount of primary surface area. Fins are commonly applied for heat management in electrical appliances such as computer power supplies or substation transformers. Other applications include IC engine cooling, such as fins in a car radiator. It is important to predict the temperature distribution within the fin in order to choose the configuration that offers maximum effectiveness.

This exercise serves as a visualization tool for evaluating the effect of shape on fin effectiveness, efficiency, and temperature distribution.

2 Modeling Details

The computational domain is represented in two dimensions.

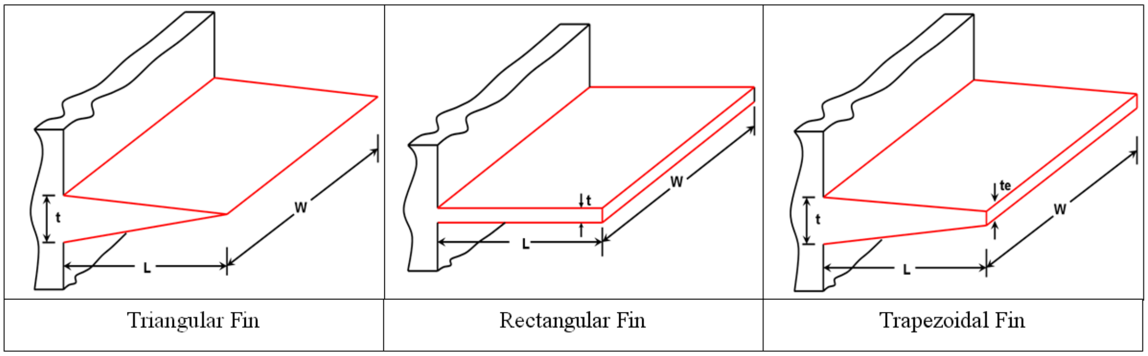
The procedure for solving the problem is:

- Create the geometry.
- Set the material properties and boundary conditions.
- Mesh the domain.

FlowLab creates the geometry and mesh, and exports the mesh to FLUENT. The boundary conditions, material properties, and surrounding properties are set through parameterized case files. FLUENT solves the problem until either the convergence limit is met, or the number of iterations specified by the user is achieved.

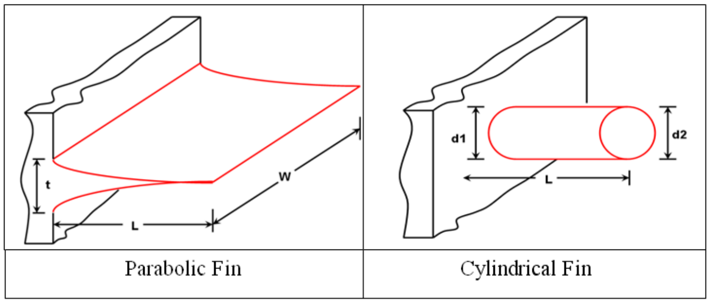
2.1 Geometry

Five different fin geometries are available. Geometric dimensions to be specified for each fin type are shown in Figure 2.1 and Figure 2.2.



Triangular Fin | Rectangular Fin | Trapezoidal Fin

Figure 2.1: Schematic Diagrams for Triangular, Rectangular, and Trapezoidal Fins



Parabolic Fin | Cylindrical Fin

Figure 2.2: Schematic Diagrams for Parabolic and Cylindrical Fins

2.2 Mesh

Coarse, medium, and fine mesh types are available. Mesh density varies based upon the assigned Refinement Factor.

Mesh Density	Refinement Factor
Fine	1
Medium	1.5
Coarse	2

Table 2.1: Refinement Factor

The meshing scheme applied is based upon the following logic:

1. For the trapezoidal and cylindrical fins, the shortest edge is identified and the edge element size is calculated by dividing the shortest edge into 8, 6 or 4 elements for fine, medium and coarse mesh types, respectively. For the rectangular, triangular and parabolic fins, edge element size is calculated by dividing the base edge into 8, 6 or 4 elements for fine, medium and coarse mesh types, respectively.
2. For the trapezoidal and cylindrical fins, the edge opposite from the shortest edge is meshed using the element size calculated in Step 1.
3. The remaining edges are meshed using the cell size calculated in Step 1.

The face is meshed after the edges are discretized into intervals. The mesh for the rectangular fin is shown in Figure 2.3.

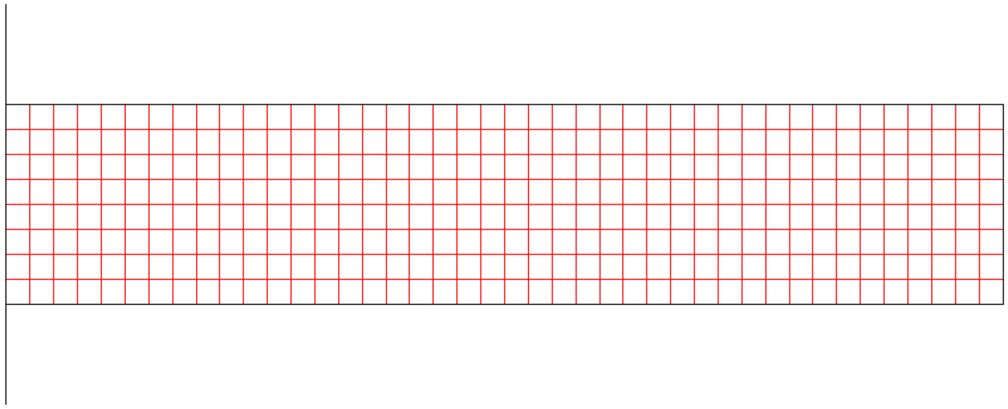


Figure 2.3: Mesh for the Rectangular Fin

2.3 Physical Models

The energy equation is solved across the solid domain. Since there is no fluid flow, mass and momentum equations are not solved.

2.4 Material Properties

The thermal conductivity of the fin material can be specified. In this exercise, the default thermal conductivity represents Aluminum. The thermal conductivity for other fin materials such as Copper, Steel, or Wood, are available. The thermal conductivity may also be specified for any solid of interest.

2.5 Boundary Conditions

The following thermal boundary conditions may be specified at the base of the fin:

- Temperature
- Heat Flux

Either fully insulated or convective type boundary conditions may be applied at the tip of the fin. For non-cylindrical fins, the fin is assumed to be infinitely wide and side surfaces of the fin (as shown in Figure 2.4) are assumed to be fully insulated. A convective type boundary condition is applied at all other walls.

2.6 Solution

The mesh is exported to FLUENT along with the physical properties and the specified initial conditions. The material properties and the initial conditions are read through the case file, and the `journal` file provides instructions for the solver to start the solution. FLUENT solves the problem until either the convergence limit is met or the number of iterations specified by the user is achieved. Upon completion, FLUENT exports the data to a `neutral` file and to `.xy` plot files. GAMBIT reads the neutral file for post-processing activities.

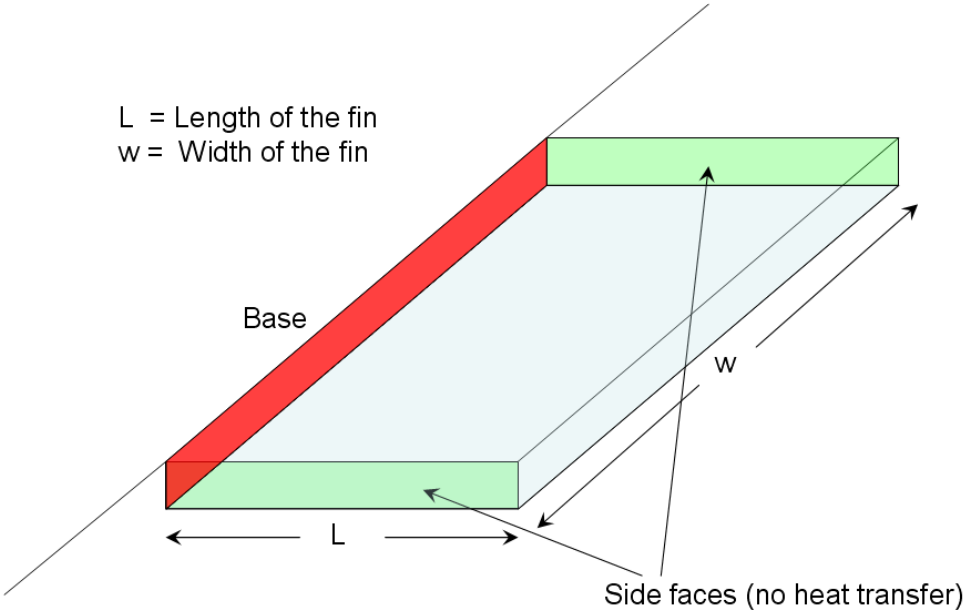


Figure 2.4: Non-cylindrical Fin

3 Scope and Limitations

The maximum temperature allowed by FLUENT (hence FlowLab as well) is 5000 K. If the temperature exceeds this limit, the temperature will be artificially restricted to 5000 K. Hence, the results obtained for cases where the temperature exceeds this limit may not be correct.

Difficulty in obtaining convergence or poor accuracy may result if input values are used outside the upper and lower limits suggested in the problem overview.

4 Exercise Results

4.1 Reports

The following reports are available:

- Average base wall temperature
- Total area for heat convection
- Total heat dissipation rate
- Max. theoretical heat dissipation rate
- Fin efficiency
- Fin effectiveness

4.2 X-Y Plots

The following plots are available:

- Residuals
- Temperature distribution

Figure 4.1 presents the temperature distribution along the length of the rectangular fin. A higher slope can be observed near the base of the fin due to the maximum temperature difference between fin surface and the surrounding medium at the base.

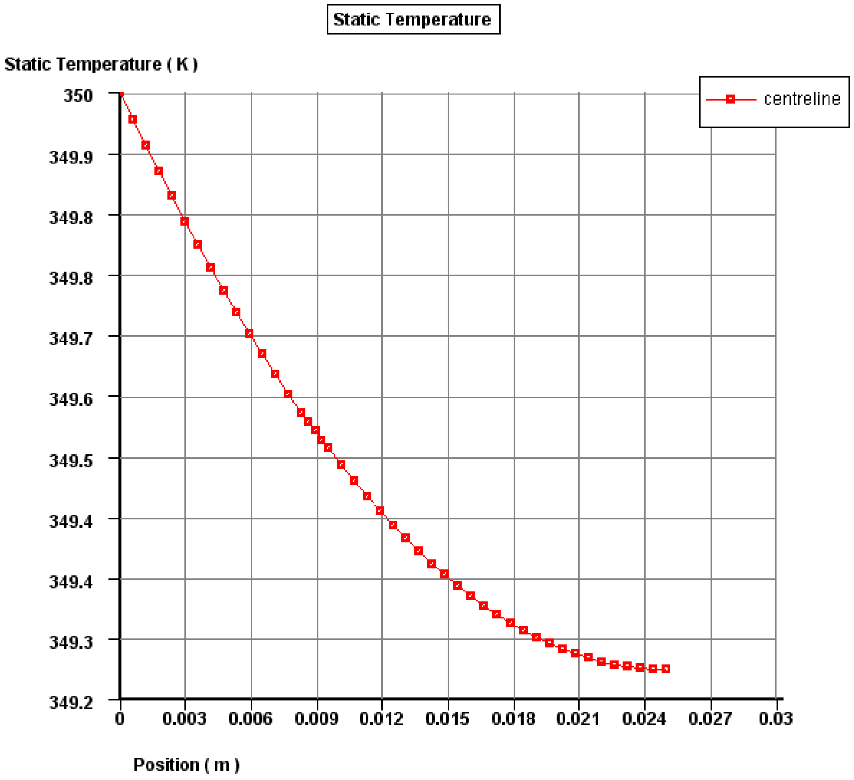


Figure 4.1: Temperature Distribution

4.3 Contour Plots

Contours of temperature can be displayed. Figure 4.2 presents temperature contours for the rectangular fin.

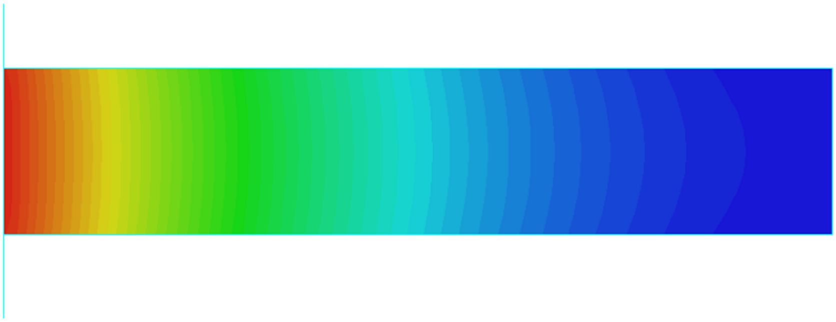


Figure 4.2: Contours of Temperature

5 Verification of Results

Table 5.1 presents results which were obtained with rectangular and triangular fin geometries using the fine mesh option and the following parameters:

Length of the fin:	75 mm
Base thickness:	3 mm
Width:	1 m
Tip condition (for rectangular fin):	convective
Fin base temperature:	573.15 K
Ambient temperature:	323.15 K
Heat transfer co-efficient:	10 W/m^2-K
Thermal conductivity:	200 W/m-K

Shape	**Fin efficiency (%)**		**Fin effectiveness**	
	FlowLab	Theory	FlowLab	Theory
Rectangular	95.99	96.14	39.36	39.418
Triangular	94.446	94.82	37.79	37.97

Table 5.1: Fin Efficiency and Effectiveness

The theoretical efficiency and effectiveness presented in Table 5.1 were predicted using the following relations:

Fin efficiency:

$$efficiency = \frac{Q_{\text{fin}}}{Q_{\text{max}}} = \frac{Q_{\text{fin}}}{hA_s\theta_0} \tag{5-1}$$

Rectangular fin:

$$Q_{fin} = \sqrt{hpkA} \times \theta_0 \frac{tanh(mL) + h/mk}{1 + (h/mk) \times tanh(mL)} \tag{5-2}$$

$$m = \sqrt{\frac{hp}{kA}},\ \theta_0 = (T_b - T_0),\ p = 2 \times (w + t) \tag{5-3}$$

Triangular fin:

$$Q_{fin} = w\sqrt{2hkt} \times \theta_0 \frac{l_1 2B\sqrt{L}}{l_0 2B\sqrt{L}},\ B = \sqrt{\frac{2Lh}{kt}} \tag{5-4}$$

where,

A = Cross-sectional area of the rectangular fin
L = Length of the fin
p = Perimeter
w = Width of the
fin t = Thickness of the fin
h = Heat transfer coefficient
k = Thermal conductivity
T_b = Base temperature
T_0 = Ambient temperature
A_s = Total convective surface
I_0 & I_1 = Bessel functions

Fin Effectiveness:

$$effectiveness = \frac{Q_{fin}}{Q_{without-fin}} = \frac{Q_{fin}}{hA_b\theta_0} \tag{5-5}$$

where,

A_b = Base area of the fin

6 Reference

[1] *Incropera, F. P., and DeWitt, D. P.*, "Fundamentals of Heat and Mass Transfer", 4th Ed.