Steady State Conduction

Abstract—*In this exercise, one-dimensional heat conduction is modeled. The solid is represented in two dimensions. The length of the solid is specified as an input. Coarse, medium, and fine meshes are available. The thermal conductivity of the solid can be specified within limits. You have the option of specifying a temperature or a heat flux at the left wall, along with a temperature at the right wall. Heat flux and temperature gradient are reported. A plot of temperature distribution is available. Contours of temperature can be displayed.*

1 Introduction

Thermal conduction is an important mode of heat transfer. Fourier's law of heat conduction relates the heat transfer rate to the temperature gradient, where thermal conductivity is represented by a constant of proportionality. One-dimensional steady state thermal conduction is modeled in this exercise.

2 Modeling Details

The solid is represented in two dimensions by a rectangle. The procedure for solving the problem is:

1. Create the geometry.
2. Set the solid properties and boundary conditions.
3. Mesh the domain.

FlowLab creates the geometry and mesh, and exports the mesh to FLUENT. The boundary conditions and flow properties are set through parameterized case files. FLUENT continues to solve the problem until the convergence limit is satisfied or the specified number of iterations is achieved.

2.1 Geometry

The geometry is created from a set of four vertices. Edges are created over the vertices and are stitched with a face to facilitate meshing. The geometry consists of four walls (Figure 2.1).

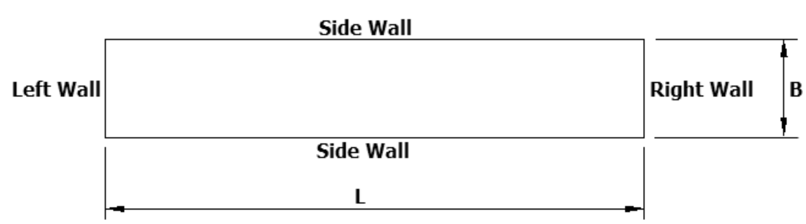

Figure 2.1: Schematic of the Solid with Boundaries

You have to specify the length (L) of the domain. The breadth (B) of the domain is calculated using the formula B = 0.1 x L.

2.2 Mesh

Coarse, medium, and fine mesh types are available. The discretization scheme used is based on the following logic:

- `$S1` = Number of cells along the length, L.
- `$S2` = Number of cells along the edge, B.

The numerical values for the two discretization parameters are given in Table 2.1.

Mesh Type	$S1	$S2
Coarse Mesh	30	3
Medium Mesh	40	4
Fine Mesh	50	5

Table 2.1: Mesh Discretization Logic

The face is meshed using the map scheme after the edges are discretized into intervals (Figure 2.2).

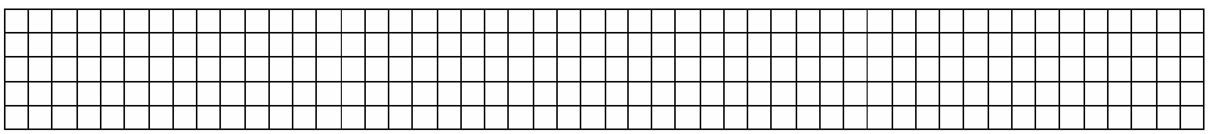

Figure 2.2: Mesh Generated by FlowLab

2.3 Physical Models for FLUENT

The energy equation is solved in FLUENT for the solid domain. Since there is no fluid flow, mass and momentum equations are not solved.

2.4 Material Properties

The default solid material is Aluminum. The thermal conductivity of the solid can be specified within limits. Other materials such as Copper, Steel, Wood, and a User Defined solid can also be selected.

2.5 Boundary Conditions

You can specify the following boundary conditions:

- Right wall temperature
- Left wall temperature *
- Left wall heat flux *

* *Based on the thermal condition (heat flux or temperature specification at the left wall).*

The following boundary conditions are assigned in FLUENT:

Left Wall	Wall
Right Wall	Wall
Side Walls	Symmetry

Table 2.2: Boundary Conditions Assigned in FLUENT

2.6 Solution

The mesh is exported to FLUENT along with the physical properties of the solid and the initial conditions specified. The material properties and the initial conditions are read through the case file. Instructions for the solver are provided through a `journal` file. Once the solution is converged or the number of specified iterations is met, FLUENT exports data to a `neutral` file and to `.xy plot` files. GAMBIT reads the `neutral` file for postprocessing activities.

3 Scope and Limitations

The maximum temperature allowed by FLUENT (hence FlowLab as well) is 5000 K. If the temperature exceeds this limit, it will be artificially restricted to 5000 K. Hence, the results obtained for cases where the temperature exceeds this limit may not be correct.

Difficulty in obtaining convergence or poor accuracy may result if input values are used outside the upper and lower limits suggested in the problem overview.

4 Exercise Results

4.1 Reports

The following reports are available:

- Heat flux
- Average temperature gradient

4.2 XY Plots

The plots reported by FlowLab include:

- Residuals
- Temperature distribution

Figure 4.1 presents temperature versus axial position.

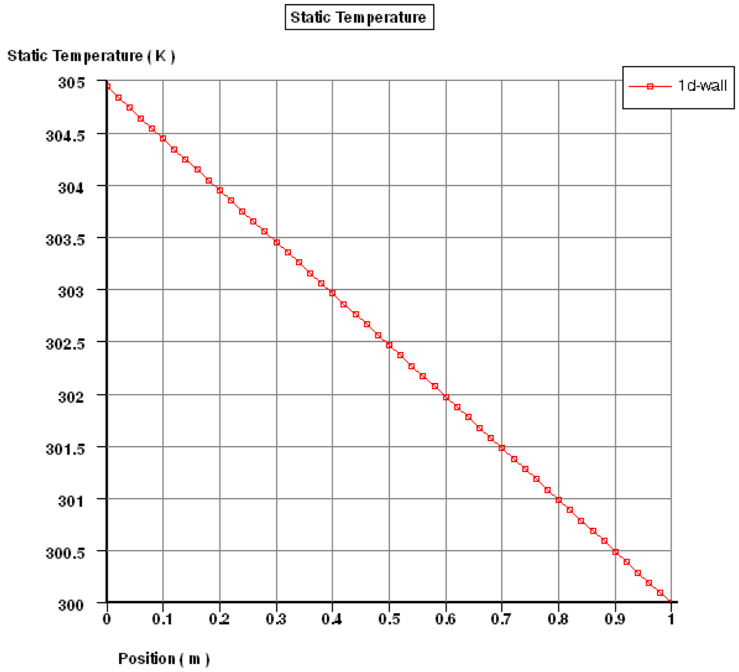

Figure 4.1: Temperature Distribution through the Solid

4.3 Contour Plots

A contour plot of temperature is available (Figure 4.2).

Figure 4.2: Contour Plot of Temperature

5 Verification of Results

For a material of thermal conductivity k and of length L, the heat flux at steady state in is given by:

$$q_x = \frac{k(T_1 - T_2)}{L} \quad \text{(5-1)}$$

where T_1 and T_2 are the temperatures at the ends of the geometry.

The temperature gradient is given by:

$$\frac{dT}{dx} = \frac{q_x}{k} \quad \text{(5-2)}$$

The temperature gradient reported by FlowLab for a thermal conductivity of 202.4 $W/m-K$ and length of 1 m is compared with the theoretical value in Table 5.1. The temperature of the right wall is maintained at 300 K. The results were generated using the fine mesh option and a convergence criterion of `1.0e-10`.

Heat Flux	**Temperature Gradient**	
	FlowLab	**Theory**
1000	4.94055	4.94
2000	9.88126	9.88
5000	24.7034	24.70

Table 5.1: Temperature Gradient Verification Heat Flux

The results presented in Table 5.2 were generated using a specified temperature at the left wall along with the fine mesh option, a thermal conductivity of 202.4 $W/m-C$, and a length of 1 m.

Left Wall Temperature	**Heat Flux**	
	FlowLab	**Theory**
400	20240	20240
500	40480	40480
1000	141680	141680
5000	951280	951280

Table 5.2: Heat Flux Verification

6 Sample Problem

1. Run the case using the default thermal conductivity and boundary conditions provided while comparing the effect of mesh density on reported results. It can be observed that temperature gradient varies slightly with mesh density. Why? Compare the results obtained with the theoretical results.
2. Change the left wall boundary condition from heat flux to *wall temperature* and run the same default settings using all the three mesh options. Again, it can be observed that heat flux value varies slightly depending upon mesh density. Compare the results obtained with the theoretical results.

7 Reference

[1] *Incropera, F. P., and DeWitt, D. P.*, "Fundamentals of Heat and Mass Transfer", 4^{th} Ed., Ch. 3.